▨ ANOTHER INTEGRAL FROM HW 2

- In problems 1 & 4 you encounter two sorts of integrals, of the form

$$\int_{-A}^{A} dx' \frac{(x-x')}{((x-x')^2+y^2+z^2)^{3/2}} \quad \text{and} \quad \int_{-A}^{A} dx' \frac{(y \text{ or } z)}{((x-x')^2+y^2+z^2)^{3/2}}$$

The 1st can be done w/ the substitution $u = (x-x')^2+y^2+z^2$. But the 2nd is trickier. Forgetting the factor in the numerator, which does not depend on x', we need to evaluate something like:

$$\int_{-A}^{A} dx' \frac{1}{((x-x')^2+\alpha^2)^{3/2}} \qquad \text{w/ } \alpha^2 = y^2+z^2 \text{ in this case.}$$

Consider the substitution $x - x' = \alpha \tan u$.

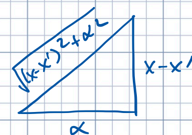
$$x - x' = \alpha \tan u \Rightarrow -dx' = \alpha \sec^2 u \, du$$

$$(x-x')^2 + \alpha^2 = \alpha^2 \times (\tan^2 u + 1) = \alpha^2 \sec^2 u$$

$$\int dx' \frac{1}{((x-x')^2+\alpha^2)^{3/2}} = \int du \, \cancel{\alpha} \, \cancel{\sec^2} u \, (-1) \frac{1}{\alpha^{\cancel{3}\,2} \sec^{\cancel{3}} u}$$

$$= -\frac{1}{\alpha^2} \int du \cos u = -\frac{1}{\alpha^2} \sin u$$

$$= -\frac{1}{\alpha^2} \underbrace{\sin\left(\tan^{-1}\left(\frac{x-x'}{\alpha}\right)\right)}_{\frac{(x-x')}{\sqrt{(x-x')^2+\alpha^2}}}$$



$$\Rightarrow \int_{-A}^{A} dx' \frac{1}{((x-x')^2+\alpha^2)^{3/2}} = -\frac{1}{\alpha^2} \frac{(x-x')}{\sqrt{(x-x')^2+\alpha^2}} \Bigg|_{-A}^{A}$$